A direct sequential cosimulation algorithm and its application in hydrogeophysics

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Introduction

Geophysical surveys often provide exhaustive maps of secondary information that can guide the interpolation of a primary variable (hydraulic conductivity for example). However, most often the relation between the variables are modeled with a linear statistical relationship (often between the log of the variables). Here, we argue that these approaches are special cases of a broader approach in which the relation between the two variables should be modelled by a joint probability density function.

An advantage of considering the joint relation in this manner is that it allows to model situations in which the conditional probabilities are multimodal. In this framework, we present a direct conditional co-simulation technique that allows creating collocated conditional simulations of the primary variable from a set of data points of the primary variable, an exhaustive map of the secondary variable, a numerical description of the joint PDF between the two variables, and a model of spatial correlation for the primary variable.

Proposed algorithm

The proposed algorithm of PDF combination proceeds as follows:

1. *Define a random path in the simulated image.*
2. *Extract the discretized conditional PDF from the joint distribution (conditional to the secondary value v_2 at the simulated node), and rescale it to obtain $P_{cond}(v_1)$.*
3. *Compute the local mean (m_0) and standard deviation (s_0) by ordinary kriging.*
4. *Discretize the PDF of kriging $G(m_0,s_0)$ and rescale it to obtain $P_{krig}(v_1)$.*
5. *Combine $P_{cond}(v_1)$ and $P_{krig}(v_1)$ to obtain the combined PDF $P_{combi}(v_1)$. Two combination approaches are possible:*
   a) *Multiply both PDFs: $P_{combi}(v_1) = P_{cond}(v_1)\,P_{krig}(v_1)$. This what we call **probability conjunction** (Tarantola 2005).*
   b) *Use the **tau model** (Krishnan 2005 , see below).*
6. *Draw a value from $P_{combi}(v_1)$ and attribute it to the simulated node.*
7. *Add this node to the data set.*
8. *Iterate until all nodes are simulated.*

The tau model

The tau model is more sophisticated than probability conjunction, as it goes beyond the hypothesis of data independence. This method allows combining partially redundant information. Redundancy is quantified by a factor τ that must be calibrated using synthetic cases.

According to the tau model, the combined probability $P(A)$ associated to the primary variable conditionally to secondary information $P(B_1)$ and $P(B_2)$ is given by:

$$P(A\,|\,B_i, i=1,\dots,n) = \frac{1}{1+x} \in [0,1], \qquad \text{with} \qquad x = \prod_i \left(\frac{x_i}{x_0}\right)^{\tau_i}$$

$$x_i = \frac{1-P(A\,|\,B_i)}{P(A\,|\,B_i)} \in [0,\infty] \qquad x_0 = \frac{1-P(A)}{P(A)} \in [0,\infty]$$

τ_1 is fixed to 1. τ_i has to be defined in order to weight each contributing variable according to its interaction with the other variables. In the present case, τ_2 only has to be defined.

If no τ value is available, taking $\tau_i = 1$ assumes data independance. In the test application, as no synthetic proxy case was available, we used this assumption, even if the variables are not independent. Nevertheless, this leads to better estimates than other methods.

Another method: the cloud transform

The cloud transform algorithm (Bashore et al. 1994, Kolbjørnsen & Abrahamsen 2004) is based on a Sequential Gaussian Simulation associated to Normal-Score transforms. This method is widely used in the petroleum industry. It proceeds as follows:

1. *Compute the Normal-Score transform of the primary variable data using the local conditional PDFs.*
2. *Adjust a variogram model on these transformed data.*
3. *Perform a conditional simulation using the variogram of (2) and the transformed data of (1).*
4. *Perform the Inverse-Normal-Score transform using local conditional PDFs.*

Conclusion

Assuming independence of both sources of information is clearly wrong as the kriging information is based on previously simulated points that already integrated information of the joint distribution. But we show here that even when the redundancy factor τ is unknown, assuming independence (setting $\tau_2 = 1$) gives better results than traditional method that are based on Gaussian assumptions.

Both method using PDF combination prove to be more efficient than the traditional cloud transform method. Simply multiplying both PDFs (probability conjunction) gives satisfying results. But finally, the best results are given by the PDF combination using the tau model, even if the redundancy factor is unknown.

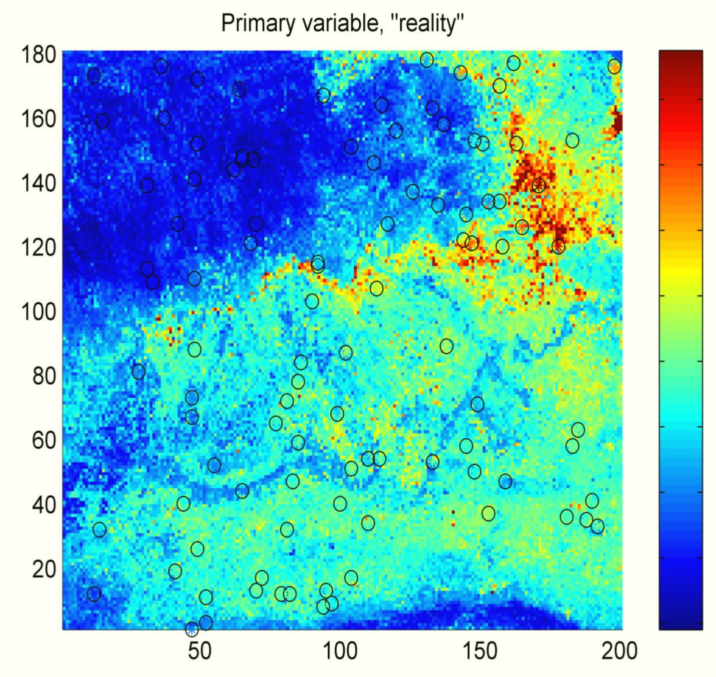
Spatial cross-correlations are neglected at this stage of the work, but they could be integrated in the simulation algorithm.

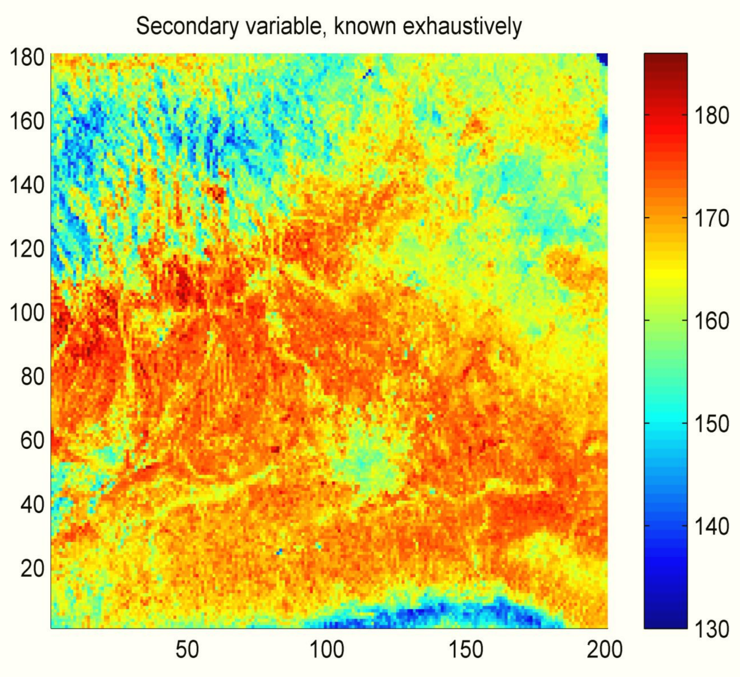
Test application

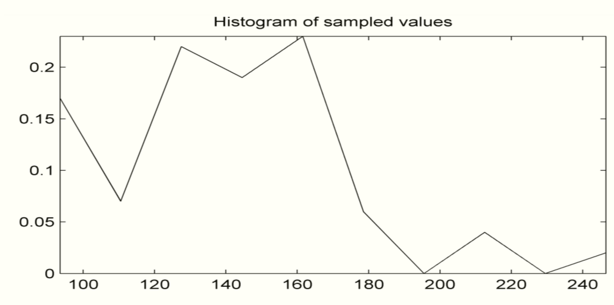
The approach is illustrated with data from satellite images of Cyprus taken at various wavelengths (photos from the USGS Landsat project).

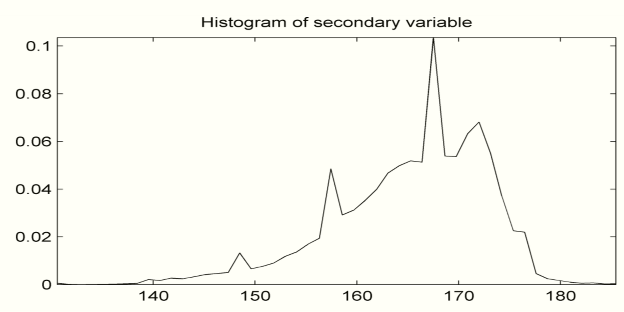
The images of the primary and the secondary variables are two satellite photos from the same area taken at different wavelengths. Each wavelength highlights different features of the land surface. However, there is a correlation between both photos, that can be expressed as a joint distribution. This joint distribution is known.

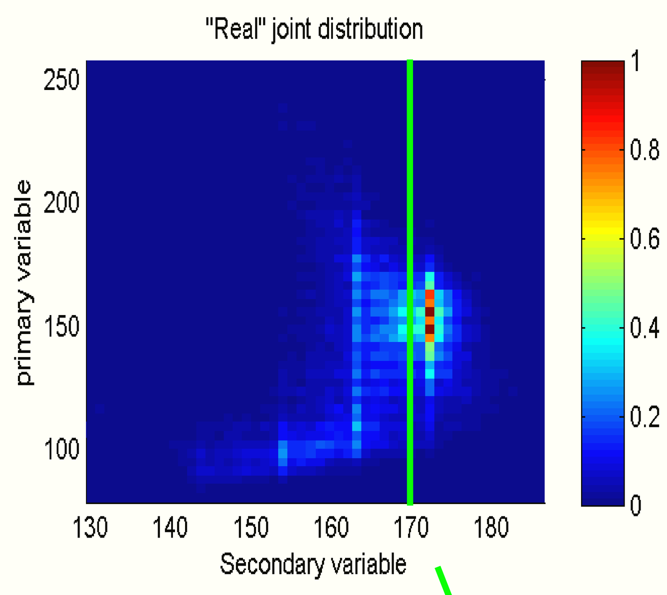
Our goal is to obtain realisations that are as close as possible from the "real" primary variable.

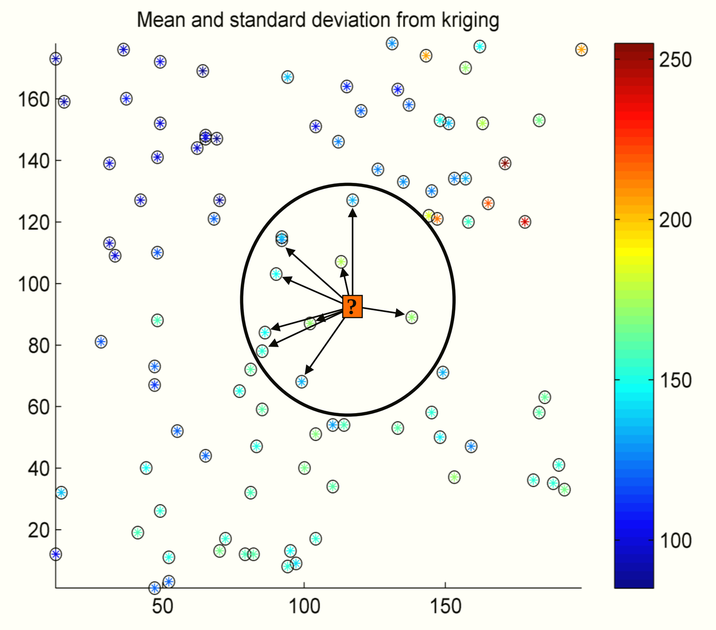


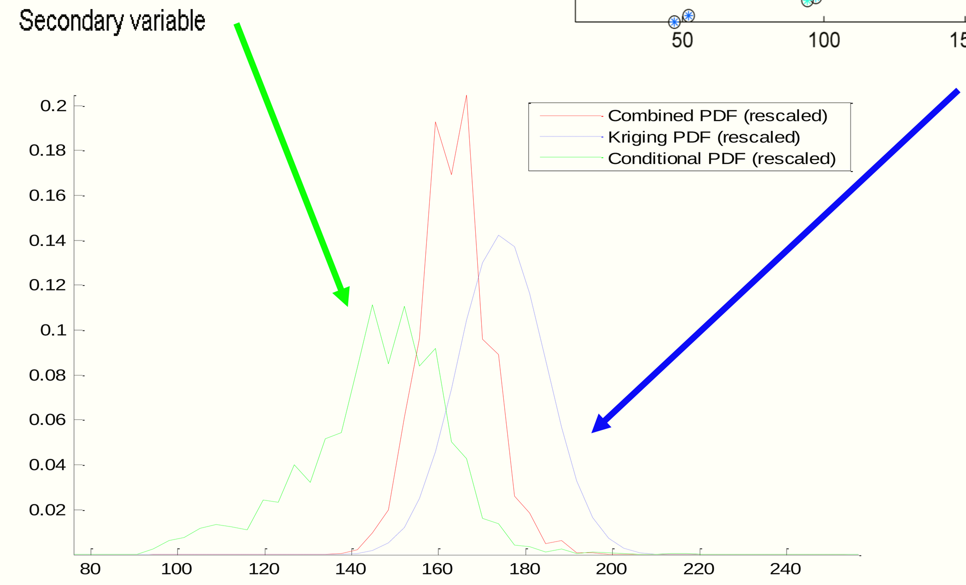






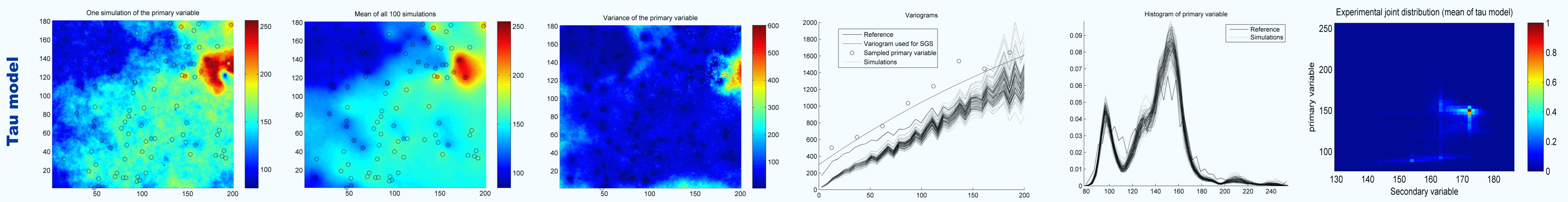






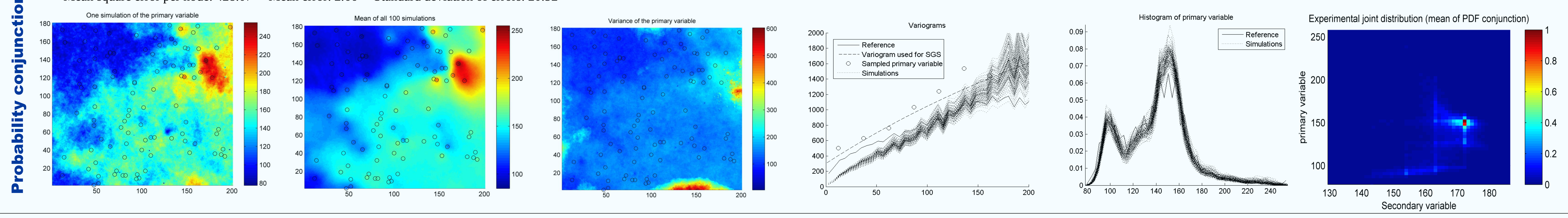
Mean square error per node: 386.96 Mean error: 1.92 Standard deviation of errors: 19.58 $\tau_2 = 1$

Tau model



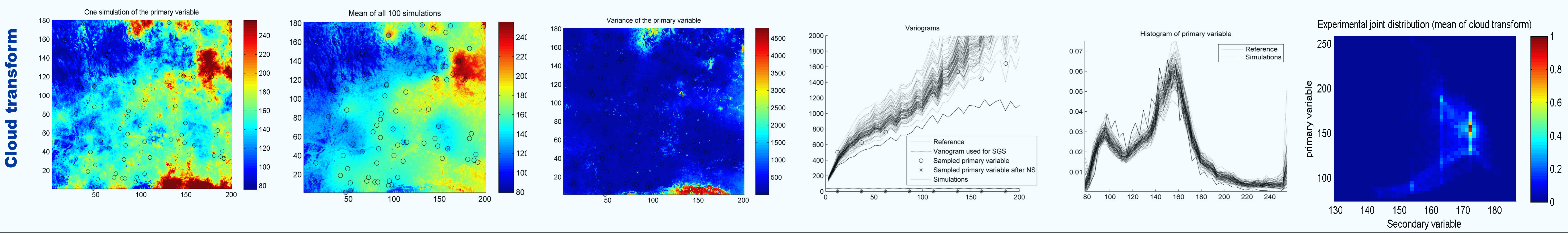
Mean square error per node: 428.07 Mean error: 2.66 Standard deviation of errors: 20.52

Probability conjunction



Mean square error per node: 914.38 Mean error: 8.58 Standard deviarion of errors: 29.00

Cloud transform



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