

# A comparison of six inverse methods for modelling 2D steady-state groundwater flow and mass transport in mildly to strongly heterogeneous synthetic aquifers

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Several inverse modelling methods have been developed during the last decades, but hardly any comparisons among them have been published. We present a comparison of the performance of six inverse methods, the Regularized Pilot Points Method (both in its classical estimation (RPPM-CE) and the Monte Carlo (MC) simulation (RPPM-CS) variant), the Monte-Carlo variant of the Representer Method (RM), the Sequential-Self Calibration method (SSC), the Zonation Method (ZM) and a Semi-Analytical Method (SAM).

The aforementioned methods were tested on a two-dimensional synthetic set-up, depicting the steady-state groundwater flow around a well. Their relative performances were assessed in terms of characterisation of (a) the reference log-transmissivity field, (b) the reference hydraulic head distribution and (c) the well catchment delineation. Simulations were performed for a mildly and strongly heterogeneous transmissivity field (i.e.  $\sigma_{\ln T}^2 = 1.0$  and  $\sigma_{\ln T}^2 = 5.3$ ). Adopted comparison measures include the absolute mean error, the root mean square error and the average ensemble standard deviation (whenever a method allows evaluating it) of the log-transmissivity and hydraulic head fields. In addition, the estimated median and reference well catchments were compared and the uncertainty associated with the estimate was evaluated. We found that, in both the mildly and the strongly heterogeneous cases, MC based methods (RPPM-CS, RM and SSC) yield very similar results in all tested scenarios, despite they use different parameterization schemes and different objective functions. The linear correlation coefficient between the estimates obtained by the different MC methods increases with the number of stochastic realisations and attains values up to 0.99 for 500 stochastic realisations. For the mildly heterogeneous case, the other inverse methods (i.e. non MC) yielded results which were consistent with those obtained by MC based inverse models. Non MC methods were negatively affected by strong heterogeneity. However, their performance was also good and the differences between all methods were not very large.