GROUNDWATER FLOW IN POROUS MEDIA

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Glossary

Aquifer: A geological formation or structure that stores and transmits water.
Darcy's Law: An empirically derived equation for the flow of fluids through porous media.
Flow equation: Mathematical law governing fluid behavior in a porous medium.
Hydrogeology: Compendium of Earth Sciences devoted to the study of groundwater, with particular emphasis on its quality and quantity.
Life Support System: A life support system can be defined as a natural or human engineered system that furthers the life of the biosphere in a sustainable fashion. The fundamental attribute of life support systems is that, jointly considered, they provide all of the needs for continuance of life.
Porous medium: Portion of space occupied by heterogeneous matter (solid matrix, gas and/or liquid and void space). Moreover, a minimum number of voids must be interconnected, allowing fluid to flow.
Sustainability: Management practices that do not take more from a groundwater system than it can provide. Theoretically, sustainable management practices can continue in perpetuity, since they do not lead to exhaustion of natural resources. For example, the removal of water from an aquifer in excess of recharge is, in the long term, an unsustainable management method, called overexploitation.

Summary

A life support system can be defined as a natural or human engineered system that furthers the life of the biosphere in a sustainable fashion. The fundamental attribute of life support systems is that, jointly considered, they provide all of the needs for continuance of life. Groundwater is an important life support system, as it represents an important reservoir for life in the biosphere.

The aim of this article is two-folded. On the one hand, authors would like to highlight the importance of groundwater systems, as well as to persuade the readers to be careful about decisions concerning them, as water is a scarce and necessary resource that should be protected and preserved.

On the other hand, groundwater flow is a very important research topic. Thus, authors would like to remark that this article sets up only the basic concepts for further reading, making special emphasis on groundwater quantity and quality. More specialized research should require the checking of the suggested bibliography.

1. Introduction

Groundwater can be defined as the water stored underground in rock crevices (fractured media) and in the pores of geologic materials (porous media) that make up the earth’s crust. Nowadays, groundwater is considered as an important life support system, both in terms of human water supply and sustenance of some ecosystems. Wrong decisions about groundwater management can provoke undesired non-reversible effects, concerning quality and quantity. These concepts are strongly linked to groundwater motion.

Flow (movement of groundwater) in porous media is a common research topic, encountered in many branches of engineering, groundwater hydrology, etc. Obviously, this article cannot include everything related to this topic. Its objective is to present the basic principles of groundwater flow in porous medium and to set up a starting point for future studies.

This article is organized as follows. First, porous medium is defined and different classifications are presented. Section 3 shows the basic parameters controlling groundwater flow and the principles that will provide the base to establish the partial differential equations used to characterize groundwater motion. The latter will be presented in section 4. Work ends a simple example illustrating the basic principles and some remarks about the status of this research topic.

2. The porous medium. Water reservoirs

The concept of porous medium is not only used in Hydrogeology (compendium of Earth Sciences devoted to the study of groundwater, with particular emphasis on its quality and quantity). Examples of porous medium are numerous: soils, fissured rocks, even a filter paper, etc. In a hydrogeological framework, and depending on the type of medium, we will talk about fractured media (fractured or fissured rocks, where water flows mostly through fractures) or porous media (sands, gravels, etc., where water flows through pores) as will be discussed later.

The common characteristics among different types of porous medium allow to setting up a standard definition: portion of space occupied by heterogeneous matter (solid matrix, gas and/or liquid phase and void space). Moreover, a minimum number of voids must be interconnected, allowing fluid to flow. Otherwise, fluid would be trapped in isolated voids.

An aquifer or groundwater basin is a geologic formation that contains water and is capable of purporting significant amounts of water, being this operation economically feasible. This amount of water moves through the porous medium, flowing in the void space, fissures, fractures, etc. In this work, only flow of water in non-fractured media will be considered. Moreover, only the part of the aquifer filled with water (saturated zone, i.e., pores completely filled with water) will be taken into account.

This definition of aquifer is somewhat subjective, as it is based on economic terms, given that to obtain a large amount of groundwater for industry supply can be as important as to obtain a small quantity to supply an arid region.

The opposite concept is the aquiclude, a geologic formation that contains water (even in considerable amounts), but is incapable to transmit it, and so that, it is not susceptible of being exploited.

On the other hand, an aquitard is a semi impervious geologic formation that transmits the water at a very slow rate. So that, it is not capable of being directly exploited.

The most common aquifers are unconsolidated geologic formations, such as sands, gravels, etc., with different origins (fluvial, sedimentation processes, erosion, etc.) (See *Typical Hydrogeological Scenarios*).

The basic tool to understand the behavior of geologic formations transmitting (or not) groundwater is the well. It is not much more than an excavation, generally vertical and cylindrical in form and often walled in, drilled to such a depth as to penetrate the geologic material of study. If the latter is water yielding, well should allow the water to be pumped to the surface. In any case, it allows the chance of taking measurements of groundwater depths.

Aquifer classification can be established in terms of the hydrostatic pressure (a close inspection of Figure 1 will help in the understanding of the above basic concepts):

- Unconfined or free aquifers, presenting a water table at atmospheric pressure (phreatic level). The water level at a well connected to this kind of aquifer is the same as the water table outside the well, under no pumping conditions. In this aquifer, one can find a “real” surface of water (the so called water table). When water table reaches earth surface, natural sources, wetlands, lakes, etc., appear. These systems are clear proofs of the existence of an important relationship between groundwater and surface water (see *Groundwater-Surface water relationships*).

- Confined aquifers, containing water between two relatively impervious boundaries. The water level in a well tapping a confined aquifer stands above the top of the confined aquifer and can be higher or lower than the water table that may be present in the material above it. In some cases, the water level can reach the ground surface, yielding a flowing well. In this case, one cannot talk about a “real” surface of water or water table, as water is only visible at a well tapping the confined aquifer. In this case, the correct term is piezometric level, as will be shown in next section.

- Leaky aquifers, as a particular case of confined aquifers, where the confining stratum is an aquitard, permitting a very low water percolation from/to other connected formations.

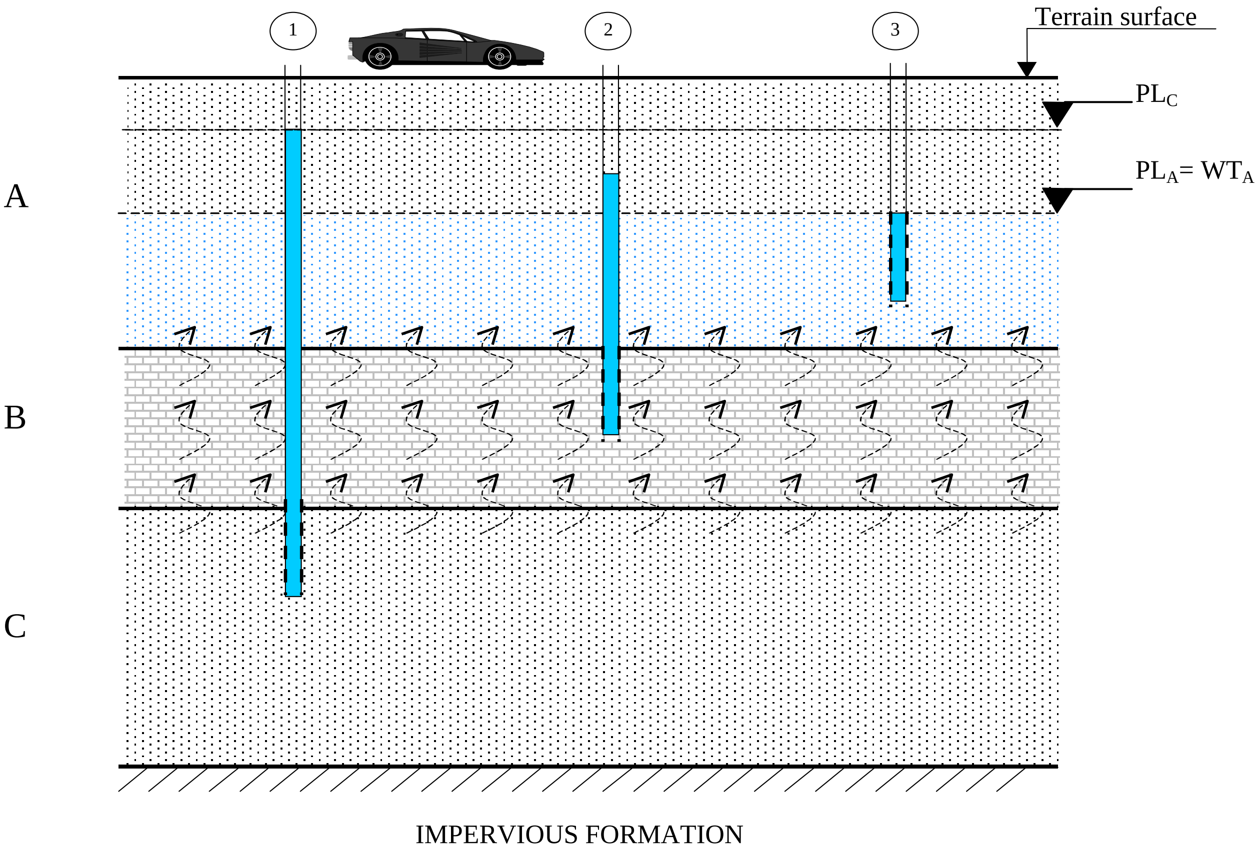


Figure 1: Classification of aquifers in terms of hydrostatic pressure (no pumping conditions)

A) Unconfined/free aquifer. Water table denoted by WT_A. It coincides with the limit of the saturated zone of aquifer A.

B) Confining formation of aquifer C. Aquitard presenting flow from formation C to formation A (dashed arrows), because piezometric level in aquifer C is larger than piezometric level in aquifer A.

C) Leaky (semi-confined) aquifer with head level PL_C. If formation B is an aquiclude (impervious formation, so there is no flow between formations C and A), then formation C is a confined aquifer. Notice that PL_C could reach terrain surface.

1) Well tapping formation C. Water height at well corresponds to PL_C. If PL_C reaches terrain surface, this well is also known as "flowing well".

2) Well tapping formation B. Water height at well is an intermediate value.

3) Well tapping formation A. Water height at well corresponds to PL_A.

Figure 1 still deserves further explanations:

1) Wells are opened only at their innermost part, and this zone is depicted with a dashed thick line. If they were opened in a larger extension, the well will mix the water of several formations.

2) Upper formation is a free aquifer, with a real water surface, depicted as WT_A (water table, aquifer A). Under no pumping conditions, well number 3, tapping this aquifer maintains water level at the water table level.

3) The correct term for the real or virtual water surface (for unconfined or confined aquifers, respectively) is piezometric level (denoted by PL in the picture).

4) One can find different piezometric levels in the same hydrogeological scenario (e.g. PL_C and PL_A in the system depicted in Figure 1).

Once the relationship between groundwater and the medium has been presented, the basic principles controlling groundwater motion can be set up.

3. Basic principles of groundwater flow in porous media

The microscopic study of a porous medium is extremely complex, given the complicated shape of the pores and the fanciful disposition of the flow paths. Fortunately, some macroscopic properties can be established, treating the porous medium as a continuum with well-defined average properties.

These properties are fully characterized by three parameters: permeability, porosity and storativity. Darcy’s law establishes the fundamental macroscopic relationship and sets up the starting point to formulate the partial differential equation that controls groundwater flow.

3.1 Dynamics of fluids in porous media

The fluid occupying the pores can be characterized by a pressure p, in such a way that in a vertical tube contacting the aquifer (piezometer), the height L of the water column (assuming equilibrium pressure) will be:

$$\mathbf{L} = \frac{\mathbf{p}}{\gamma} \tag{1}$$

where γ is the specific weight of the fluid ($\gamma=g\cdot\rho$, where g is gravity and ρ is fluid density).

As mentioned before, water depth can be measured in a well, but also in a piezometer (similar to a well but, generally, with a smaller diameter, and considered just for measurement, not for water-pumping purposes).

Considering a given reference horizontal plane, the water level (piezometric head level) in the piezometer is (see Figure 2):

$$\mathbf{h} = \mathbf{z} + \frac{\mathbf{p}}{\gamma} \tag{2}$$

where z is the height of the point with respect to the reference plane (this plane or height reference is oftenly taken to be equal to the mean water sea level). One can observe that head level defines the energy of the water per unit weight. For instance, the first term “z”, can be expressed as potential energy (m·g·z, being m fluid mass) per unit weight (m·g). In a static system, all points have the same head level (but not the same pressure), as depicted in Figure 2.

Aquifer dimensions can be huge. The separation between confined or unconfined is not so strict as the previous definition may say, because its behavior can vary in space. Figure 2 shows this change. On the left and right zones (depicted in blue), aquifer behaves as a free aquifer. Therefore, one could observe a “real” surface of water (water table). Once the aquifer is over-pressured (zone marked with dashed line), aquifer behaves as confined. However, if the system is static, the water table/piezometric level (depending on the zone) remains unchanged within the whole aquifer (free or confined) extension. Therefore, piezometric levels do not change and the aquifer is under no flow conditions.

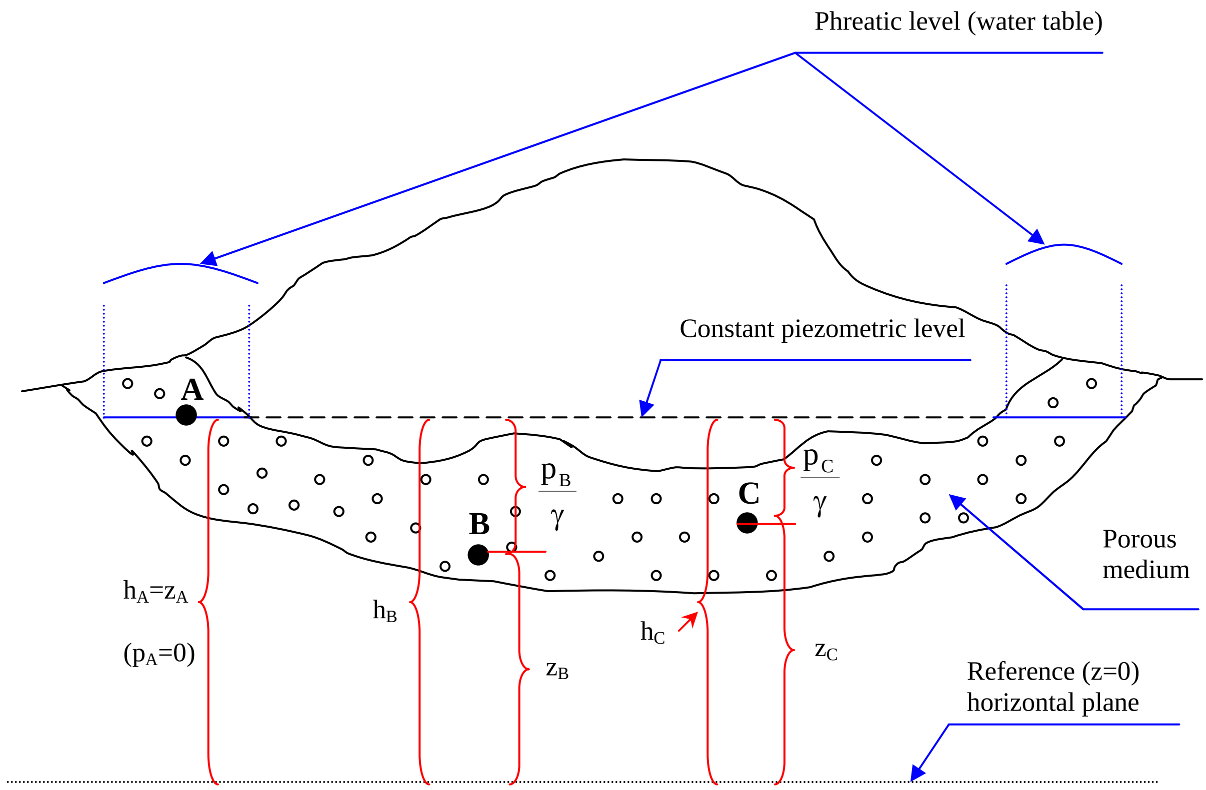


Figure 2. Aquifer under no flow conditions (static system). Observe that points A, B and C have the same head level, but a different pressure. Point A has atmospheric pressure (taken as 0).

Let us make a step further to include the fluid movement. Consider for this purpose a pseudo-cylinder of porous medium, as indicated in Figure 3, with a fluid moving through it. Bernouilli's equation can be written for two points A, B, in the same flow path and separated by a distance d_{AB}:

$$\frac{p_A}{\gamma_A} + z_A + \frac{v_A^{*2}}{2g} = \frac{p_B}{\gamma_B} + z_B + \frac{v_B^{*2}}{2g} + \Delta h_{AB} \tag{3}$$

where v^* is the real velocity of the fluid, Δh_{AB} is the head loss between A and B (due to frictions) and γ is the specific weight of the fluid. Again, all terms in (3) can be viewed as energy per unit weight (pressure, potential and kinetic energy). The hydraulic gradient between A and B is:

$$\mathbf{grad\,h} = \nabla h \approx \frac{\Delta h_{AB}}{d_{AB}} \tag{4}$$

being d_{AB} the distance between A and B, following a flow line connecting A and B. A common mistake, while calculating hydraulic gradients is to consider the geometric distance between A and B, instead of using the distance that the fluid particle has to cover to reach point B from point A (i.e. distance along a flow path).

Usually, groundwater velocities are negligible, so that (3) can be expressed as:

$$\frac{p_A}{\gamma_A} + z_A = \frac{p_B}{\gamma_B} + z_B + \Delta h_{AB} \tag{5}$$

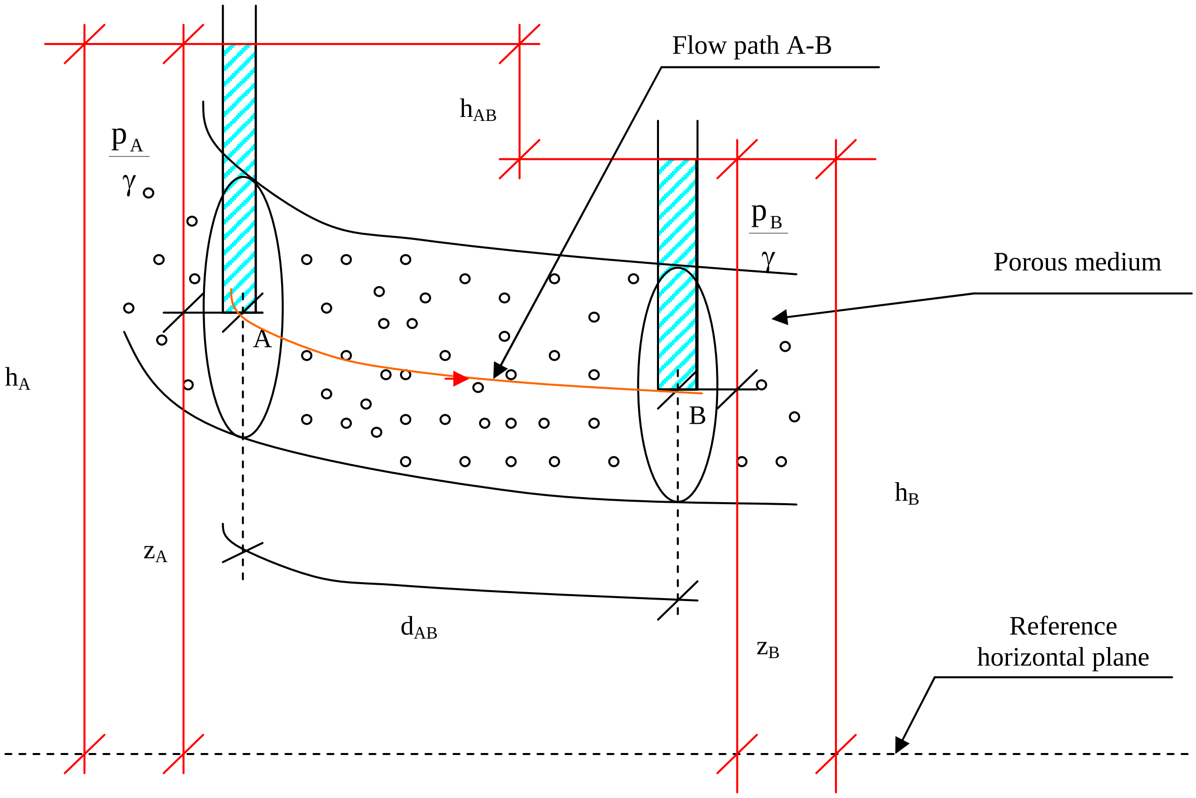


Figure 3. Hydraulic system in a porous medium with fluid motion from A to B (from higher to lower head levels).

All fluids move from higher to lower energy states. In a porous medium, water starts moving when a head variation exists and always from a high-energy state (high head level) to a lower energy state (low head level). Another common mistake is to consider that fluids move from high to low pressure zones. This affirmation is only valid for gases, where position energy (z) is negligible against pressure energy (p/γ). Notice that water can move from low to high-pressure zones, if the head loss is positive.

3.2 Darcy's law. Permeability.

In 1856, Henry Darcy stated the foundation stone for several fields of study, including groundwater hydrology, soil physics, and petroleum engineering. It is an experimental relationship between groundwater discharge flow rate (fluid flux) and the flow area, elevation, fluid pressure and a proportionality constant.

Darcy's law states that the hydraulic (head level) gradient in equation (4) can be obtained through:

$$v = -k\nabla h \qquad (6)$$

where v is the fluid flux (volume of fluid per unit surface per unit time) and k is a coefficient, called permeability. Fluid flux v is also denoted as Darcy's flux, specific flux or Darcy's velocity. This law was experimented with cylinders filled with different porous materials and has been tested continuously by different investigations. However, the latter expression has some restrictions, discussed later.

Permeability is determined through Darcy's experiment (see Figure 4). Water is forced to go through a cylinder of porous media, by fixing head level at both left and right hand side. Therefore, the hydraulic gradient and distance between A and B are known a priori (notice that the flow direction of water is forced to be parallel to the axis of the cylinder).

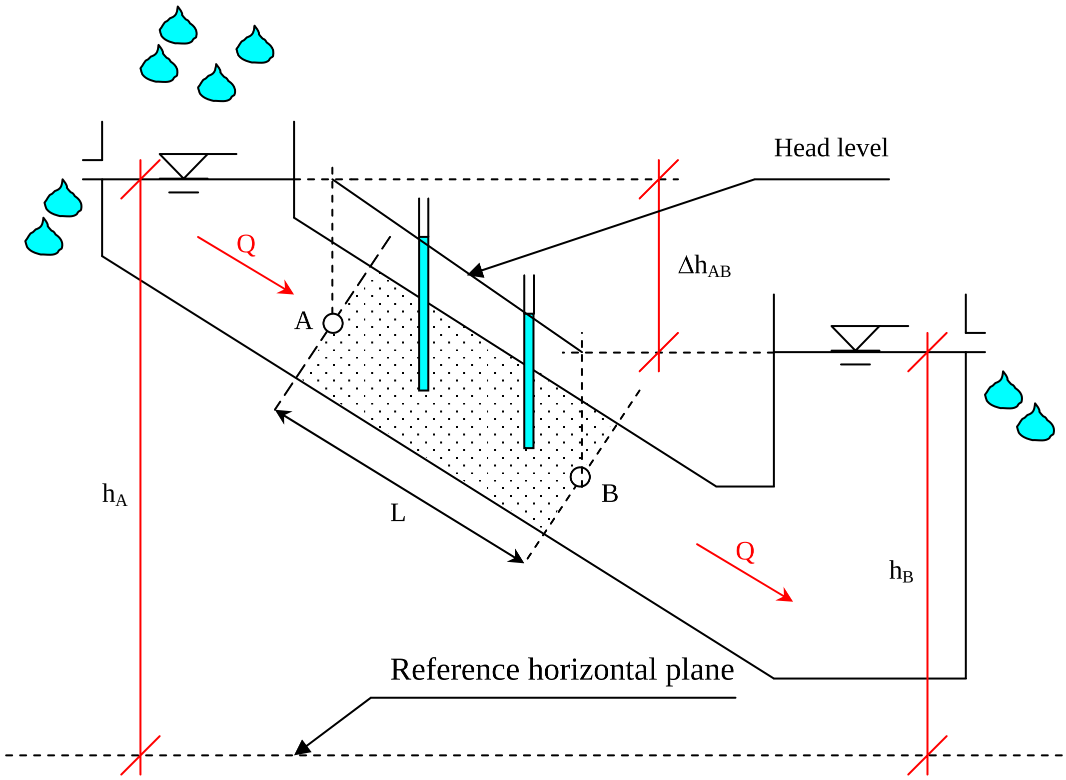


Figure 4: Darcy's experiment design. Water moves from A to B (higher to lower energy).

Head loss is also known and, in this case:

$$\nabla \mathbf{h} = \frac{\Delta \mathbf{h}}{\mathbf{L}} \tag{7}$$

so that

$$\mathbf{v} = -\mathbf{k}\nabla \mathbf{h} = \mathbf{k}\frac{\Delta \mathbf{h}}{\mathbf{L}} \tag{8}$$

The circulating flow rate is also known:

$$\mathbf{Q} = \mathbf{vA} = \mathbf{kA}\frac{\Delta \mathbf{h}}{\mathbf{L}} \tag{9}$$

where A is the cross section of the cylinder. Therefore, permeability (also known as hydraulic conductivity) can be easily calculated as:

$$\mathbf{k} = \frac{\mathbf{QL}}{\Delta \mathbf{hA}} \tag{10}$$

Permeability can be defined as the property of the medium, expressing the relative ease with which fluids can pass through, but it also depends on the fluid itself (e.g., water flows easier than petroleum, due to viscosity phenomena).

Permeability measurements are complicated, as the influence of the granulometry and the composition of the material is high. Moreover, spatial variability of permeability is huge (concept of heterogeneity, see *Flow and transport in highly heterogeneous aquifers*), and the measured value can vary depending on the considered direction (concept of anisotropy, see *Groundwater flow in clay-rich aquifers*). For instance, the permeability in a fracture plane is orders of magnitude higher than the permeability in the orthogonal direction (flow along fracture direction is easier than flow through rock matrix).

An additional problem is the size of the material being tested (scale of the problem). The measurements obtained through field experiments are not the same as the measurements obtained at the laboratory. At a field scale, measurement is an average value, with high uncertainty (depending on the size of the tested zone). At the laboratory, test does not respect regional conditions, working at small scale.

The traditional way to obtain the permeability values is through field experiments, but it can be also obtained through laboratory experiments (via permeameter-tests, reproducing more or less Darcy's experiment) or following empirical expressions that correlate permeability with another soil properties (easy to measure), such as granulometry. The latter is not a good policy, as all proposed formulations are inferred on the base of a specific-site data.

Permeability values for some materials are presented in Table 1. These values should be considered as a first approach and prone to error, as said before.

Table 1. Orientation values of permeability and relationship between aquifer and material types (modified from Custodio et al., 1983)

Permeability (m/day)	**10^{-7}-10^{-4}**	**10^{-4}-10^{-2}**	**10^{-2}-10^{-0}**	**10^{-0}-10^{3}**	**10^{3}-10^{4}**
Classification	Impervious	Very low permeable	Low permeable	Permeable	Highly permeable
Aquifer type	Aquiclude	Aquitard	Poor aquifer	Good aquifer	Excellent aquifer
Material type	Compact clay Shale Granite	Sandy silt Silt Silty clay	Coarse sand Silty sand Karstic limestone	Sandy gravel Coarse sand	Coarse gravel

3.3 Darcy's law limitations

Darcy's law establishes a macroscopic property of the medium, given that a large number of pores are considered. Equation (6) can de expressed as:

$$\nabla \mathbf{h} = -\frac{\mathbf{v}}{\mathbf{k}} \tag{11}$$

The latter expression is valid only for small velocities (laminar flow). This is the most common case when considering flow in a porous medium. For large velocities (turbulent flow), gradient increases much faster than velocity (flow in a fracture or in a karstic medium). In this case, equation (10) becomes:

$$\nabla \mathbf{h} = \mathbf{b}\mathbf{v}^2 \tag{12}$$

where b is a constant of proportionality.

Flow regime (laminar/turbulent) can be defined through Reynolds number:

$$\mathrm{Re} = \frac{v d \rho}{\mu} \tag{13}$$

where v is Darcy's flux ($cm \cdot s^{-1}$), ρ is fluid density ($g \cdot cm^{-3}$), μ is the dynamic viscosity ($dina \cdot s \cdot cm^{-2}$), hat can de defined as the resistance of a fluid to flow. d (cm) is an average particle size (the size of the sieve capable of retaining fifty per cent of the material, for unconsolidated materials).

Darcy's law is valid only if the number of Reynolds is under a threshold value (in fact, flow is completely turbulent for Re>60 to 180, being all of the suggested thresholds dependent on the material type).

$$\mathrm{Re} < 4 \text{ to } 10 \tag{14}$$

3.4 Total and effective porosity

A porous medium was defined as a grain aggregate, with a fluid occupying the intergranular space. The voids are the so-called pores. Given a volume V of porous medium, where $V_m=V-V_{pt}$ is the volume of compact matrix (V_{pt} is then the total volume occupied by the pores). Total porosity is defined as:

$$\phi_t = \frac{V_{pt}}{V} \tag{15}$$

While studying groundwater flow, only interconnected pores are relevant, because only the water from the connected pores can be removed. The definition of effective porosity can be stated in a similar way, but considering only interconnected pores (V_p):

$$\phi_{eff} = \frac{V_p}{V} \tag{16}$$

Total porosity can be measured in laboratory (through gravimetric, volumetric, optic or nuclear methods) or in-site, by interpretation of geophysical data, obtained through well-logs interpretation.

Orientation values for some materials are presented in Table 2. Reader must not consider this values as standard, as they are prone to error, due to local peculiarities, such as meteorization (degradation of the material, due to physical, chemical or biological processes -erosion, clogging-, etc.), precipitation/dissolution processes, material age and consolidation, etc.

3.5 Water velocity

Water flows across the porous medium with variable velocity, depending on the size, shape and pores disposition; again, at pore scale, water velocity cannot be evaluated but, as well as Darcy's velocity, it can be treated as a macroscopic property, if the considered volume is large enough.

Table 2. Total and effective porosity values for some materials (modified from Custodio et al., 1983).

Material		**Total porosity % (ϕ_t)**	**Effect. porosity % (ϕ_{eff})**
Type	Description	Average	Average
Massive rocks	Granite	0.3	<0.2
	Massive limestone	8	<0.5
	Dolomite	5	<0.5
Metamorphic rocks		0.5	<0.5
Volcanic rocks	Pyroclasts	30	<5
	Volcanic ashes	25	20
	Basalt	12	5
Consolidated sedimentary rocks	Shales	5	<2
	Sandstone	15	<2
	Detritic limestone	10	3
Unconsolidated sedimentary rocks	Alluvial basins	25	15
	Sands	35	25
	Non-cohesive clays	45	2
	Soils	50	10

A common error is to confuse the real velocity of the water and Darcy's velocity. The latter does not take into account that medium is a grain-aggregate (all paths are valid to flow). That is not true, as water flows through pore-paths (interconnected pores). However, both quantities are related to each other, through the concept of effective porosity (ϕ_{eff}).

$$\mathbf{v}^* = \frac{\mathbf{v}}{\phi_{\mathbf{eff}}} \tag{17}$$

where v^* is the average real velocity of the water and v the Darcy's velocity. Notice (see Table 2) that effective porosity is a fraction of total porosity and it can be orders of magnitude smaller.

3.6 Recharge

How can water be obtained from an aquifer? The usual way is by pumping. A pump is put inside the well and the water is pumped out to the surface and then distributed. It is then relevant to know the exact amount of water that can be extracted from an aquifer.

A basic parameter for this purpose is the recharge. Recharge is the amount of water that enters into an aquifer per unit time. Usually, the main source of recharge is the precipitation. However, frequently only a small fraction of precipitation reaches the aquifer. Mostly, precipitation rambles out over surface.

As for the precipitation, recharge is generally expressed as an areal recharge, that is, as volume of water per surface unit (it has usually, length units, most times mm). It is really important to know the recharge of an aquifer, because this is the maximum amount of water that can be extracted from the aquifer within sustainable limits. If it is extracted (per unit time) more water than recharge for a long time period (it is said that the aquifer is overexploited), the aquifer losses water and several unliked situations may be reached: it can be dried (so no more water is available), it can be too expensive to pump, because piezometric level is decreasing, so water must be pumped from a lower location, natural discharges of the aquifer to a life support system (e.g. wetland) may be reduced or even disappeared, etc. Those are some of the reasons why it is important to know accurately the recharge. However, a problem comes when one tries to evaluate it. As many other aquifer parameters, it is very difficult to evaluate. There are no direct evaluations. All the ways are indirect evaluations from other parameters. One common way is by means of a water balance in the soil. Without going deeply into details, some of the factors that are taken into account in such a balance are: precipitation (the main source of recharge), evaporation, transpiration (water extracted from the soil by plants) and water retained by capillary forces, among others. Recharge evaluation is difficulted by other aspects. In a regional aquifer (aquifer with a large extension), the recharge may be produced very far away from the study zone (Figure 5).

Recharge is a natural process. However, it can be also produced in a synthetic way, the so-called artificial recharge (man induced recharge). That is done in specific contexts, for example to use the aquifer as a reservoir, to use the soil as part of a depuration process, etc.

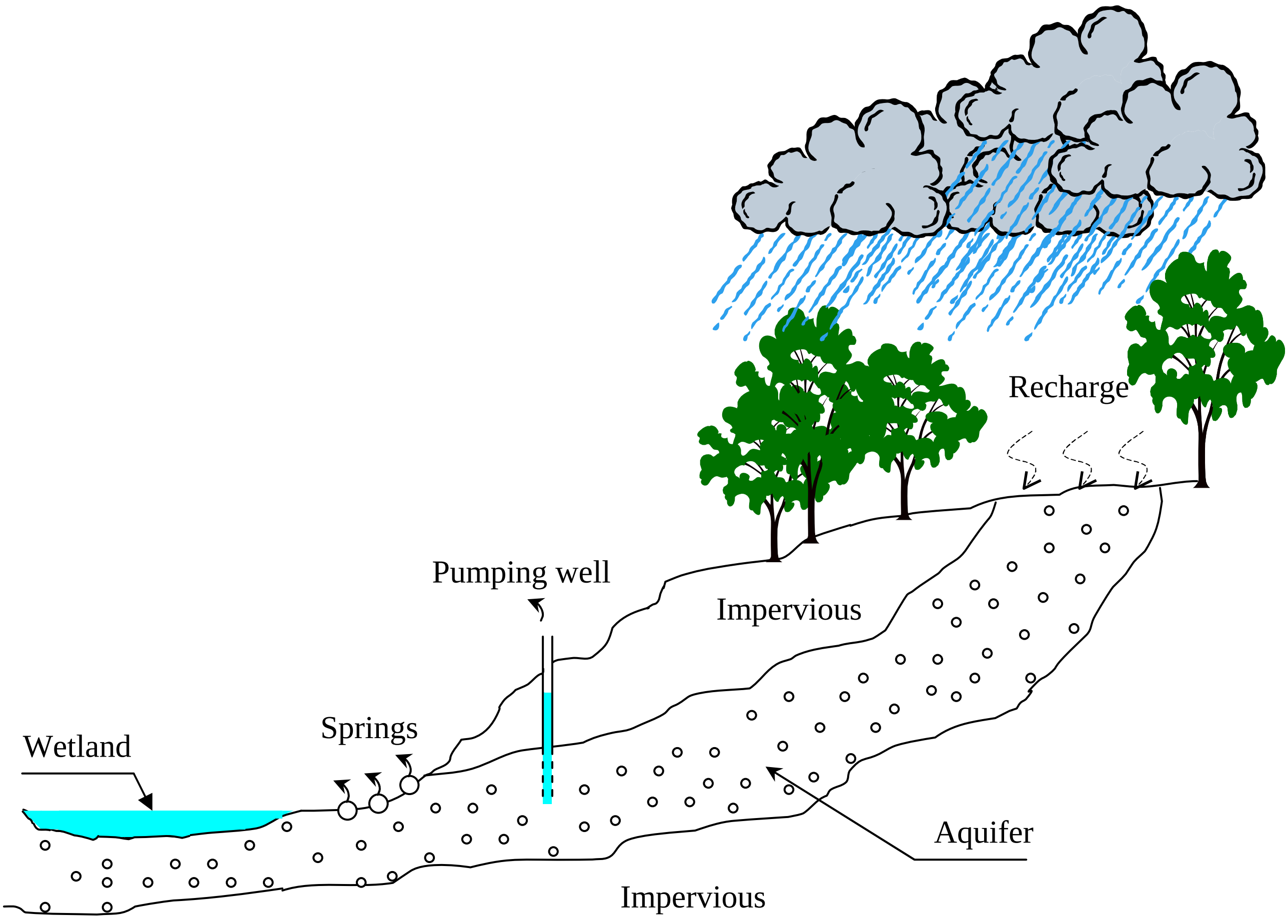


Figure 5. Prior to man action, the system was in equilibrium (this is the so-called steady state, i.e., the amount of recharge was equal to the discharge in the wetland and the springs). Observe that recharge takes place far away from the source and the wetland. If man action is introduced, e.g., water is pumped, the amount of water flowing to wetland and springs will be reduced. In the extreme case, this could lead to the disappearance of both wetland and springs.

3.5 Storativity

Permeability controls the ease that water finds to move in a porous medium. Recharge is the amount of water that enters the system. But, how much water can be obtained from an aquifer? The answer is given by the storativity, S. The storage coefficient is the amount of water that can be obtained in a square prism of 1 m^2 base if the piezometric level is reduced in 1 meter (see Figure 6). In case that there is no porous medium, e.g., in a lake, we obtain of course 1 m^3. In a free aquifer, storativity coincides with drainable porosity (smaller than total porosity, because some amount of water is trapped in non-connected pores and cannot be taken out). In a confined aquifer, it describes the compressibility of the confined medium. It is important, of course, to know the storativity, because it may lead to an idea of the amount of water available in the aquifer, in absence of external sinks or sources, such as pumping or recharge.

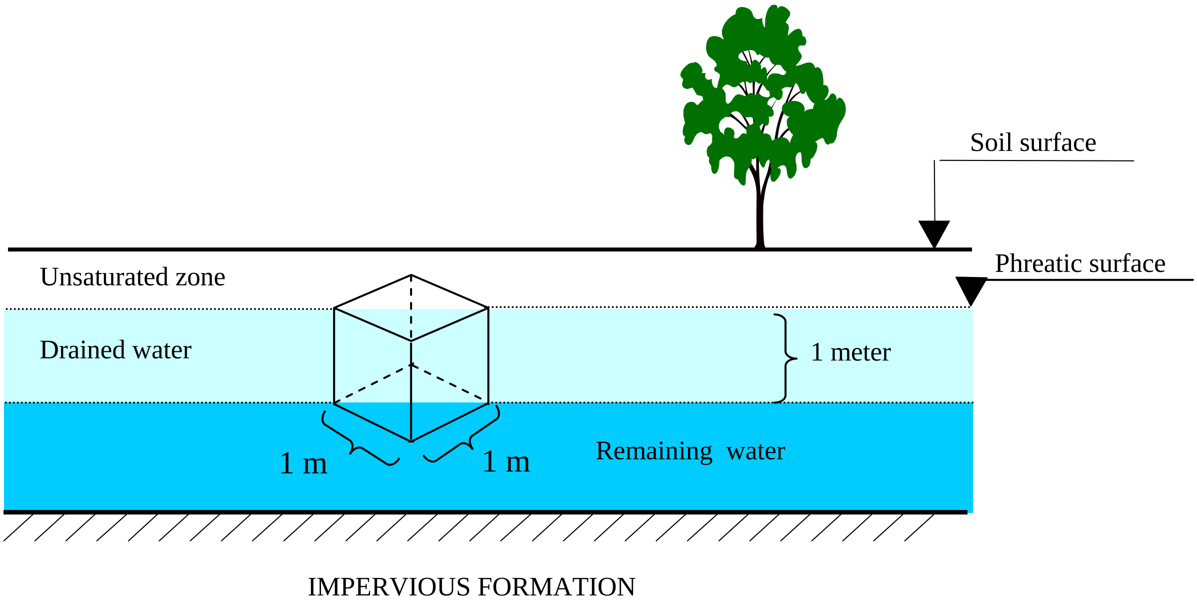


Figure 6. Storativity. If head level is reduced by 1 m in an area of 1 m^2 (depicted prism), an amount of S m^3 of water is obtained.

4 Flow equation

The flow equation is derived in the next sections. It is presented first in its steady state form, that is, assuming that the situation does not vary with time. Later it is extended to the general transient case (i.e., head level varies with time). This is the most common situation, as conditions affecting head level usually vary along time (due to variations in recharge, pumping, etc.).

4.1 Continuity equation

The departing point is the fact that the incoming mass of water in a specified aquifer volume is equal to the out coming mass in steady state. It is also assumed a continuity hypothesis. This hypothesis looks at the aquifer as a continuum like air, for instance (in this approach microscopic variations are averaged, it is then a good approximation from a macroscopical point of view). The way of deducing the equation is departing from a given volume (see figure 7) and looking at the flow through it. For simplicity it will be presented in two dimensions.

In the face A a mass of

$$M(x)=\rho(x)\, v_x(x)\ \ \Delta y\ \Delta t \tag{18}$$

is entering the volume in a time interval Δt, where $M(x)$, $\rho(x)$ and $v_x(x)$ are the mass in the area of figure 7, the density of fluid and the velocity in the x direction (horizontal direction) at point x, respectively.

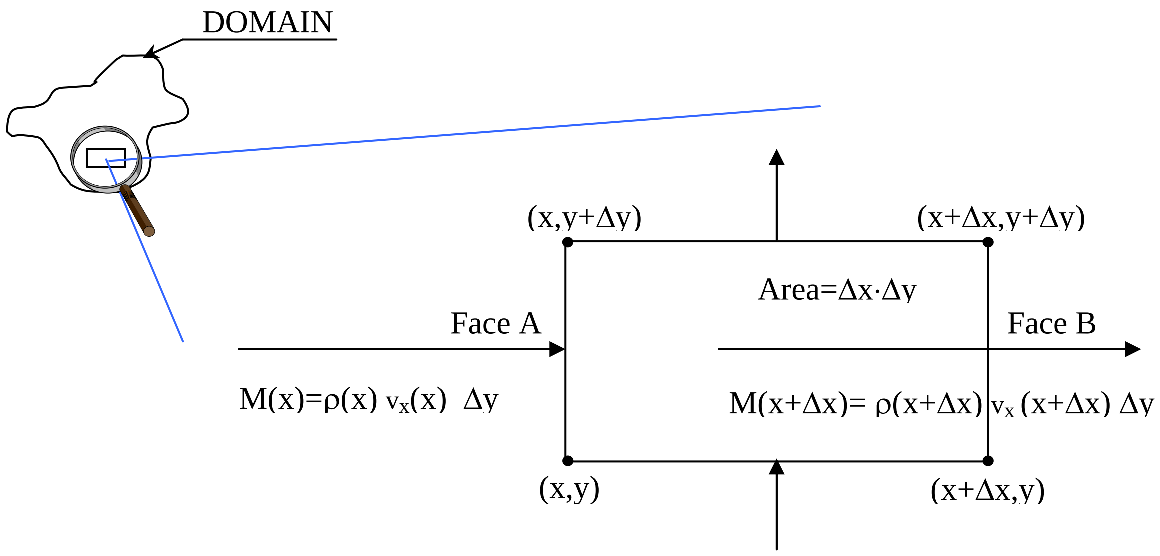


Figure 7. Representative volume (area in this case, for simplicity). A water mass balance is established inside the area.

In the opposite face, B, a mass of

$$M(x+\Delta x)=\rho(x+\Delta x)\, v_x(x+\Delta x)\ \ \Delta y\ \Delta t \tag{19}$$

is leaving the volume, where $M(x+\Delta x)$, $\rho(x+\Delta x)$ and $v_x(x+\Delta x)$ are again the mass, density and velocity in the x direction at point $x+\Delta x$, respectively. It is assumed of course, that the volume is small enough to assume that ρ and v_x do not depend on y (vertical direction) neither in side A nor B.

The mass variation between both faces is

$$\Delta M_x=M(x)-M(x+\Delta x)=[\ \rho(x)\, v_x(x) - \rho(x+\Delta x)\, v_x(x+\Delta x)\ ]\ \Delta y\ \Delta t \tag{20}$$

The term in brackets is an approximation of the derivative (the smaller the volume, the more accurate the approximation). So it may be written as

$$\Delta \mathbf{M}_x = -\frac{\partial(\rho \mathbf{v}_x)}{\partial \mathbf{x}} \Delta \mathbf{y} \Delta \mathbf{x} \Delta \mathbf{t} \tag{21}$$

This is the mass variation between faces A and B in the area of figure 7 and in a time interval Δt. So mass variation per unit distance and unit time is

$$\Delta \mathbf{M}_x = -\frac{\partial(\rho \mathbf{v}_x)}{\partial \mathbf{x}} \tag{22}$$

This reasoning can be repeated for the other axis leading to a similar expression. Adding the mass variation in both directions (x and y) it is obtained:

$$\Delta \mathbf{M} = -\frac{\partial(\rho \mathbf{v_x})}{\partial \mathbf{x}} - \frac{\partial(\rho \mathbf{v_y})}{\partial \mathbf{y}} \tag{23}$$

This expression can be written in a compact mathematical form as

$$\Delta m = - \operatorname{div}(\rho\ \mathbf{v}) \tag{24}$$

where v is velocity array, $v = (v_x, v_y)$ and div is divergence operator. If a fluid F is generated per unit area and unit time in the area of Figure 7, then (input minus output plus generated mass should be equal to zero in steady state):

$$\Delta m + \rho\ F = 0 \tag{25}$$

Joining equations (24) and (25):

$$- \operatorname{div}(\rho\ \mathbf{v}) + \rho\ F = 0 \tag{26}$$

Usually ρ is constant in space, so it can be taken out of the derivative in equation (26) and the expression can be divided by ρ, resulting

$$-\operatorname{div}(\mathbf{v}) + F = 0 \tag{27}$$

This expression has been deduced for a small area inside the whole domain (see Figure 7). If now all the equations for all the small volumes of the whole domain are summed up, and Darcy’s law is employed (see equation 6), it is possible to obtain flow equation (in steady state) for the whole domain as

$$\operatorname{div}(\mathbf{K} \operatorname{grad} h) + q = 0 \tag{28}$$

where **K** is conductivity, h is piezometric head and q is the sum of all the fluid mass externally generated in the whole domain (the sum of all F terms of equation (27) for all the small volumes). If there are no external mass inputs in the system, the flow steady state equation is then:

$$\operatorname{div}(\mathbf{K} \operatorname{grad} h) = 0 \tag{29}$$

4.2 Transient equation

Deduction of flow equation in transient state is quite similar to the steady state case. It also establishes a fluid mass balance. The only difference stands on the existence of an additional term in the balance (the new term depicted in red):

Input – Output + External inputs = Temporal variation inside the domain (30)

So the difficulty stands on the quantification of the new term. To do this, let us remind the definition of storage coefficient. It is the amount of water that can be obtained if the piezometric head is reduced in 1 m in an area of 1 m^2.

Thus, the variation of water (ϑ) in an arbitrary area Δx Δy and per unit time Δt is

$$\vartheta = \Delta x \Delta y S \frac{\Delta h}{\Delta t} \tag{31}$$

where S is storage coefficient, Δh is the piezometric head variation in time interval Δt. Expressing equation (31) per unit surface, taking equation (30) and joining it with steady state flow equation (28), transient flow equation is obtained:

$$S \frac{\partial h}{\partial t} = \mathrm{div}(K\,\mathrm{grad}h) + q \tag{32}$$

Piezometric head of a given aquifer can be obtained by solving this equation with the appropriate boundary and initial conditions. In general, this equation cannot be directly solved analytically and numerical methods have to be employed. The objective of this article is not to discuss about the difficulties arising while solving the equation, but let us mention that, to solve it, it is necessary to know the parameters of flow equation that were mentioned previously and that they are difficult to obtain, as only indirect measures are available (to make thing worst, those measurements are prone to error).

5 Example

The objective of this section is to present a very simple example to fix some of the key concepts presented in previous sections.

Consider an alluvial sandy aquifer of cubic form with homogeneous flow. Using the information coming from several boreholes and wells, it is known that the dimensions of the aquifer are 200 meters width (until impervious borders are reached), the aquifer thickness (depth to an impervious formation) is 30 meters, and the saturated zone is 25 meters deep (head level) in one cross-section (see Figure 8).

There is one well, 50 meters away from the cross-section, where head level is 25.25 meters. Interpretations of pumping tests stated that the permeability in the aquifer is 100 m/d (this is an average value).

Which is the flow across the cross section, if we assume that the permeability is homogeneous in the whole aquifer?

Hydraulic (head level) gradient (see Equation 4) can be approximated as:

$$\mathrm{grad}h \approx \frac{\Delta h_{AB}}{d_{AB}}$$

where the position and head level at points A and B have to be known (see Figure 8). Moreover, A and B have to be in the same flow line. In this case, considering B as the well far away from the cross-section and A a well in the cross-section (assuming that the geometrical distance between the points is approximately the same as the distance along the flow path):

$$\mathrm{grad}h \approx \frac{\Delta h_{AB}}{d_{AB}} = \frac{25 + 25.25}{50}\ \frac{m}{m} = -0.005$$

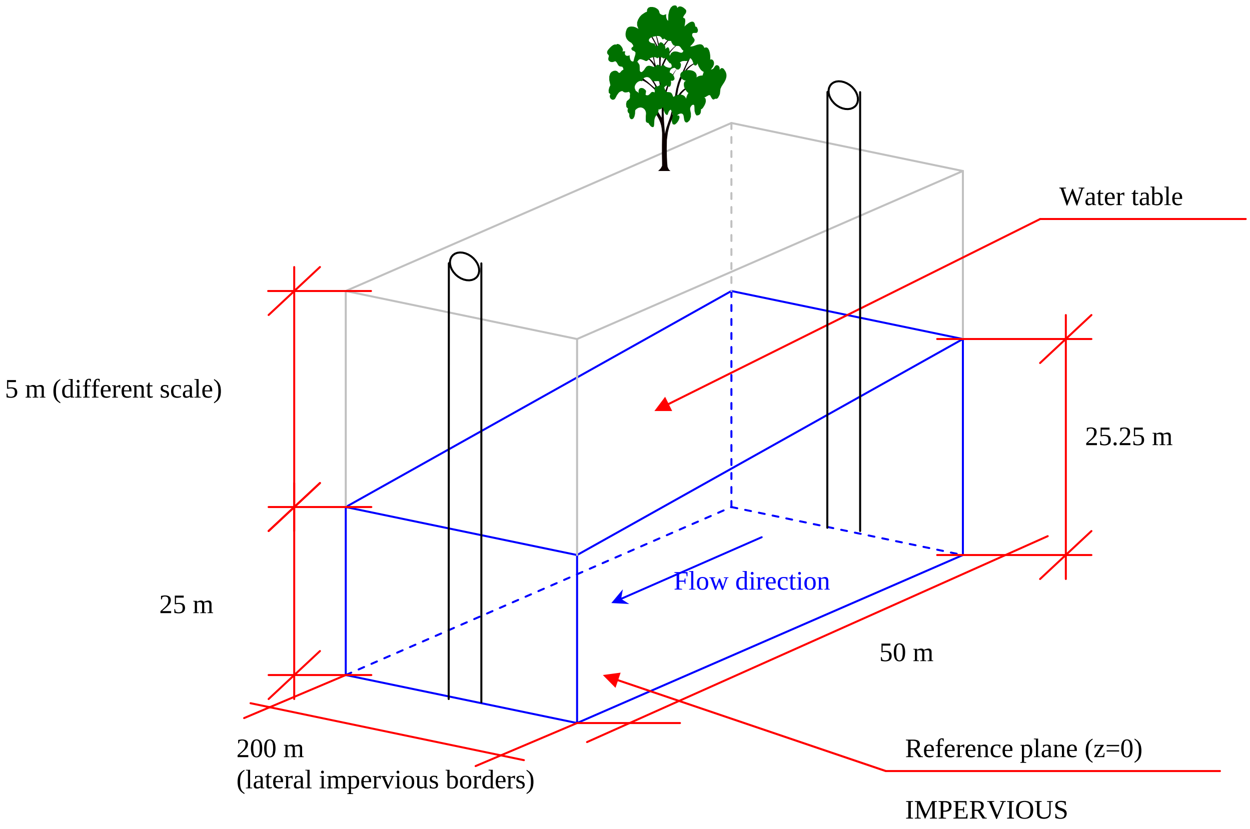
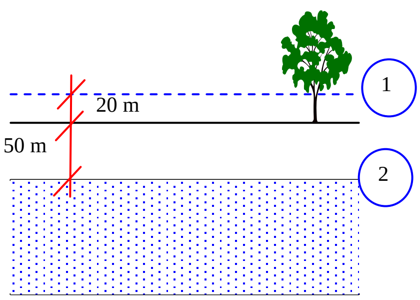


Figure 8. Example setup

Direct use of Darcy's law (see Equation 9) leads to:

$$\mathbf{Q = vA = -kgradhA \approx -100\,\frac{m}{d}\,(-0.005)\,25m\,200m = 2500\,\frac{m^3}{d}}$$

The same aquifer, but 10000 meters downstream, becomes confined, with a surface of 130 km^2. Average permeability and storativity were obtained through pumping tests (10 m/d and $5\cdot10^{-5}$, respectively), being its average thickness 40 meters. The top of the aquifer (contact with the confining impervious formation) was found at 50 meters depth. Initial head level was found at 20 meters over the land level (that means that any well in this zone will be flowing). Which volume of water could be obtained through pumping without a change in the aquifer behavior?



The volume of water that could be obtained in a rectangular prism of area 1 m^2 by decreasing head level 1 meter would be (see section 3.5),:

Unitary volume=S=$5\cdot10^{-5}$

The maximum drawdown allowed without changing the aquifer behavior (maintaining it confined) is 70 meters, distance between positions 1 and 2 (when head level reaches position 2, that matches the top of the aquifer, becomes unconfined). Therefore, the total allowed volume will be:

$$\text{Total volume}=S\ \Delta h\ A=5\cdot 10^{-5}\ 70\ \text{m}\ 130\cdot 10^{6}\ \text{m}^2=455000\ \text{m}^3$$

6. Recent advances

Groundwater flow in porous media is currently a very important research topic. In this article, authors set up only the basic concepts for further reading and highlight that there is a huge tree of different branches with the common starting point stated in previous sections.

During past decades (let us to remind that the theory presented before started in the 19-th century) and jointly with computer developments, new topics have arisen. Let us, just for the sake of completeness, to mention some of them, such as flow in "special scenarios", where the mentioned theory deserves some "peculiarities" (e.g. fractured rocks -see *Groundwater Flow through fractured rocks*-, heterogeneous aquifers -see *Flow and transport in highly heterogeneous aquifers*- or the unsaturated zone –see *Modelling Flow and Transport in the unsaturated zone*-).

As stated before, flow equation is not easy to solve analytically and, oftenly, one is forced to use numerical methods, opening a new branch in the tree: the numerical modeling of aquifers (see *Groundwater flow and transport modeling*).

A large set of disciplines also make use of this theory, as flow equation is the basic principle in groundwater modeling. New incorporations are Hydrochemistry (see *Hydrochemistry*), Hydrobiology (see *Subsurface Hydrobiology*), biology and ecology (see *Biology of Springs and Seepage zones. Ecology of groundwater discharge areas*).

Summarizing, groundwater flow rules control the quantity and the quality of the groundwater. Many different ways of studying the aquifer behavior can be used, including different disciplines. In any case, one should bear in mind that groundwater is a scarce and necessary good for the sustenance of life and should be preserved.

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